

Dynamic model of spherical perturbations in the Friedmann universe. II. retarding solutions for the ultrarelativistic equation of state

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Abstract

Exact linear retarding spherically symmetric solutions of Einstein equations linearized around Friedmann background for the ultrarelativistic equation of state are obtained and investigated. Uniqueness of the solutions in the C^1 class is proved.

1 General solution of evolutionary equation in the form of power series and private cases.

1.1 Equations of spherical perturbations model

As it is shown in Ref.[1], spherically symmetric perturbations of Friedmann metrics are described by one scalar function $\delta\nu(r, \eta)$ connected with perturbation of the metric tensor component g_{44} in the isotropic coordinates (r, η) with the relation

$$\delta g_{44} = a^2(\eta)\delta\nu, \quad (1)$$

at that the representation turns out to be convenient

$$\delta\nu = 2\frac{\Phi(r, \eta)}{ar} = 2\frac{\Psi(r, \eta) - \mu(\eta)}{ar}, \quad (2)$$

it allows to factor the solution into particlelike mode

$$\delta\nu_p = -\frac{2\mu(\eta)}{ar}, \quad (3)$$

equivalent to Newton potential of the point mass $\mu(t)$, and the nonsingular mode

$$\delta\nu_0 = \frac{2\Psi(r, \eta)}{ar}, \quad (4)$$

the “potential” of which $\Psi(r, \eta)$ and its first derivative by a radial variable at the beginning of the coordinates satisfy the relations

$$\lim_{r \rightarrow 0} |\Psi(r, \eta)| = 0, \quad \lim_{r \rightarrow 0} \left| r \frac{\partial \Psi(r, \eta)}{\partial r} \right| = 0. \quad (5)$$

By a constant coefficient of barotrope κ the mass of a particle source $\mu(\eta)$ and the nonsingular mode potential $\Psi(r, \eta)$ satisfy the evolutionary equations (75) and (84) [1]:

$$\ddot{\mu} + \frac{2}{\eta}\dot{\mu} - \frac{6(1+\kappa)}{(1+3\kappa)^2} \frac{\mu}{\eta^2} = 0, \quad (6)$$

$$\ddot{\Psi} + \frac{2}{\eta}\dot{\Psi} - \frac{6(1+\kappa)}{(1+3\kappa)^2} \frac{\Psi}{\eta^2} - \kappa\Psi'' = 0. \quad (7)$$

In the previous paper [1] an exact solution of the evolutionary equation (6) for the particlelike source mass was also obtained

$$\mu = \mu_+\eta^{\frac{2}{1+3\kappa}} + \mu_-\eta^{-\frac{3(1+\kappa)}{1+3\kappa}}, \quad (1+\kappa) \neq 0; \quad (8)$$

$$\mu = \mu_+\eta + \frac{\mu_-}{\eta}, \quad (1+\kappa) = 0. \quad (9)$$

In [1] it was noted that the peculiarity in the solution (8) by $1+3\kappa=0$ is coordinate disappearing by coming from the temporal coordinate η to the physical time t

$$t = \int a(\eta)d\eta; \Rightarrow \quad t = \bar{c}\eta^{\frac{3(1+\kappa)}{1+3\kappa}}; \quad \eta = \bar{C}_1 t^{\frac{1+3\kappa}{3(1+\kappa)}}. \quad (10)$$

Let us recall, the nonsingular part of the energy density perturbation $\delta\varepsilon(r, \eta)$ and the radial velocity of medium $v(r, \eta)$ are determined with help of the potential function $\Psi(r, \eta)$ by the relations

$$\frac{\delta\varepsilon}{\varepsilon_0} = -\frac{1}{4\pi r a^3 \varepsilon_0} \left(3\frac{\dot{a}}{a}\dot{\Phi} - \Psi'' \right), \quad (11)$$

$$(1+\kappa)v = -\frac{1}{4\pi r a^3 \varepsilon_0} \frac{\partial}{\partial r} \frac{\dot{\Phi}}{r}, \quad (12)$$

where $\varepsilon_0(\eta)$ is a nonperturbed energy density of Friedmann Universe?

$$\varepsilon_0 \sim \eta^{-\frac{6(1+\kappa)}{1+3\kappa}}; \quad a \sim \eta^{\frac{2}{1+3\kappa}}; \quad \varepsilon_0 a^3 \sim \eta^{-\frac{6\kappa}{1+3\kappa}}. \quad (13)$$

In [1] by the variables separation method a general solution of the evolutionary equation (7) satisfying the conditions (5) in form of an integral from Bessel functions was also obtained. In this paper we shall obtain more convenient solutions in form of power series.

1.2 Class C^∞ general solution in the perturbation area; $\kappa \neq 0, 1 + \kappa \neq 0$

As it was shown in the previous paper [1], the singular part of the potential function $\delta\nu(r, \eta)$ is uniquely extracted by the representation (2) in which the potential function $\Psi(r, \eta)$ is nonsingular at the beginning of the coordinates, that is satisfies the relations (5) in consequence of which, in particular

$$\Psi(0, \eta) = 0. \quad (14)$$

Further supposing in the final neighborhood $r = 0; r \in [0, r_0)$ in which the metrics perturbation is localized, the potential function $\Psi(r, \eta)$ belongs to the C^∞ class, let us represent the solution of the evolutionary equation (6) satisfying the conditions (5) in the form of power series of the radial variable r

$$\Psi(r, \eta) = \sum_{n=1}^{\infty} \Psi_n(\eta) r^n. \quad (15)$$

Let us underline the expansion (15) does not consist of a member with a zero power r in consequence of the relation (14). Substituting the function $\Psi(r, \eta)$ in the form of the Eq. (15) into the evolutionary equation (6) and setting the coefficients at each power r in the obtained equation equal to zero, we get a chain of linking equations

$$\begin{aligned} \Psi_{2m} = 0; \quad \ddot{\Psi}_{2m+1} + 2 \frac{\dot{\Psi}_{2m+1}}{\eta} - \frac{6(1+\kappa)}{(1+3\kappa)^2} \frac{\Psi_{2m+1}}{\eta^2} \\ = \kappa(2m+3)(2m+2)\Psi_{2m+3}; \quad m = \overline{0, \infty}, \quad (\kappa \neq 0, -1). \end{aligned} \quad (16)$$

Thus, by $\kappa \neq 0$ the general solution of the evolutionary equation (6) for the potential function $\Psi(r, \eta)$ is a series by odd powers of the radial variable r , that is by $\kappa \neq 0, -1$ the potential function $\Psi(r, \eta)$ of the C^∞ class is an odd function of the radial variable r

$$\Psi(r, \eta) = \sum_{p=0}^{\infty} \Psi_{2p+1}(\eta) r^{2p+1}. \quad (17)$$

For the private particle solutions responding the specific physical conditions the series can be broken at any odd $n = N \geq 3$. In this case supposing for the last member of the series

$$\Psi_n(\eta) = 0; \quad n > N = 2p + 1, \quad (p = 1, 2, \dots), \quad (18)$$

we obtain from the Eq. (16) a closed equation

$$\ddot{\Psi}_{2p+1} + 2 \frac{\dot{\Psi}_{2p+1}}{\eta} - \frac{6(1+\kappa)}{(1+3\kappa)^2} \frac{\Psi_{2p+1}}{\eta^2} = 0. \quad (19)$$

As far as this equation does not differ from the evolutionary equation for the particlelike source mass at all, its general solution will coincide with the solution (8) accurate within reterming

$$\Psi_{2p+1} = C_+^p \eta^{\frac{2}{1+3\kappa}} + C_-^p \eta^{-\frac{3(1+\kappa)}{1+3\kappa}}, \quad (20)$$

where C_+^p and C_-^p are some constants.

Substituting this solution into the next to last equation of the chain (16) we shall get the equation for determining Ψ_{2p-1} :

$$\begin{aligned} & \ddot{\Psi}_{2p-1} + 2 \frac{\dot{\Psi}_{2p-1}}{\eta} - \frac{6(1+\kappa)}{(1+3\kappa)^2} \frac{\Psi_{2p-1}}{\eta^2} \\ &= \kappa(2p+3)(2p+2) \left[C_+^p \eta^{\frac{2}{1+3\kappa}} + C_-^p \eta^{-\frac{3(1+\kappa)}{1+3\kappa}} \right]. \end{aligned} \quad (21)$$

In consequence on this equation linearity its general solution is a sum of the general solution of the corresponding homogeneous equation Ψ_{2p-1}^0 and a private solution of the inhomogeneous one Ψ_{2p-1}^1 . But the general solution of the homogeneous one coincides with the already mentioned above solution (8), and the private solution can be introduced in the form of a sum of two solutions corresponding to the two members of the right part (19). Therefore, evidently the corresponding private solution has the form

$$\Psi_{2p-1}^1 = A_{p-1} \eta^{\frac{2}{1+3\kappa}+2} + B_{p-1} \eta^{-\frac{3(1+\kappa)}{1+3\kappa}+2}. \quad (22)$$

Thus, we find

$$A_{p-1} = \frac{\kappa(1+3\kappa)}{2(7+9\kappa)}(2p+1)C_+^p; \quad B_{p-1} = -\frac{\kappa(1+3\kappa)}{6(1-\kappa)}(2p+1)C_-^p. \quad (23)$$

Substituting the obtained solutions into the previous equations let us repeat the analogues calculations/ Thus we pointed out the algorithm of constructing a general solution of an evolutionary equation (7) reduced to repeating differentiation operations. This general solution corresponding to the highest power $N = (2p+1)$ of the radial variable consists of $2N$ arbitrary constants appearing every time by solving the corresponding homogeneous differential equations.

1.3 Case N=3

Let us demonstrate the task solution in the simplest but as it turns out the most important case when $N = 3$ ($p = 1$) and the series (15) consists of only two members corresponding to the values $p = 0, 1$. In this case from Eqs. (15)-(23) we find

$$\Psi_3 = C_+^1 \eta^{\frac{2}{1+3\kappa}} + C_-^1 \eta^{-\frac{3(1+\kappa)}{1+3\kappa}}; \quad (24)$$

$$\Psi_1 = C_+^0 \eta^{\frac{2}{1+3\kappa}} + C_-^0 \eta^{-\frac{3(1+\kappa)}{1+3\kappa}} + \frac{3\kappa(1+3\kappa)}{2(7+9\kappa)} C_+^1 \eta^{\frac{2(2+3\kappa)}{1+3\kappa}} - \frac{\kappa(1+3\kappa)}{2(1-\kappa)} C_-^1 \eta^{-\frac{1-3\kappa}{1+3\kappa}}. \quad (25)$$

Let us study now the specific cases $\kappa = 0$ and $1 + \kappa = 0$ which are out of the general solution (15).

1.4 Nonrelativistic matter $\kappa = 0$

In this case we get from (8) the law of mass evolution of a particlelike source (see Ref. [4])

$$\mu = C_+ \eta^2 + C_- \eta^{-3}. \quad (26)$$

The equation (7) for the potential function Ψ takes the form

$$\ddot{\Psi} + 2\frac{\dot{\Psi}}{\eta} - 6\Psi = 0, \quad \kappa = 0, \quad (27)$$

whence we find

$$\Psi = \phi_+(r)\eta^2 + \phi_-(r)\eta^{-3}, \quad (28)$$

where $\phi_{\pm}(r)$ are arbitrary functions r . In this case the mass of a particlelike source evolves according to the law

$$\mu = \mu_+ \eta^2 + \mu_- \eta^{-3}. \quad (29)$$

1.5 Inflationary case $\kappa + 1 = 0$

In this case the equation for the field function Φ is elliptic

$$\ddot{\Phi} + 2\Phi\frac{\dot{\Phi}}{\eta} + \Phi'' = 0, \quad (\kappa + 1 = 0), \quad (30)$$

and the radial velocity of perturbation is not determined by the equation (12) which in this case gives

$$\frac{\partial}{\partial \eta} \left(\frac{\Phi}{r} \right) = 0, \quad 1 + \kappa = 0. \quad (31)$$

Integrating (31) we find:

$$\Phi = \phi(r) + \xi(\eta)r, \quad (32)$$

where $\phi(r)$ and $\xi(\eta)$ are some arbitrary functions of their arguments. Substituting this solution into the equation (31) and dividing the variables we find the function $\Phi(r)$

$$\Phi = C_1 - \frac{C_2}{2}r^3 + r \left(C_2 + \frac{C_3}{\eta} + C_1\eta \right), \quad (33)$$

where C_1 , C_2 and C_3 are arbitrary constants.

2 Retarding spherical perturbations in ultrarelativistic universe

2.1 Boundary conditions for the retarding solutions

By $\kappa > 0$ The spherically symmetric perturbations of Friedmann metrics are described by the hyperbolic equation, by $\kappa < 0$ - by elliptic one. Peculiarities of the hyperbolic equations (7) are convergent and nonconvergent waves

$$r \mp \sqrt{\kappa}\eta = \text{Const}, \quad (34)$$

spreading at the sound velocity

$$v_s = \sqrt{\kappa}. \quad (35)$$

Let us study the task for spherically symmetric solutions of linearized Einstein equations against the background of Friedmann metrics with zero boarding conditions for the potential function $\Phi(r, \eta)$ at the sound horizon corresponding to the causality principle

$$\Sigma : \quad r = r_0 + \sqrt{\kappa}(\eta - \eta_0). \quad (36)$$

In terms of the introduced functions $\Phi(r, \eta)$ and $\Psi(r, \eta)$ these conditions can be written in the form

$$\Phi(r, \eta)|_{r=r_0+\sqrt{\kappa}(\eta-\eta_0)} = 0; \Leftrightarrow \Psi(r, \eta)|_{r=r_0+\sqrt{\kappa}(\eta-\eta_0)} = \mu(\eta); \quad (37)$$

$$\Phi'(r, \eta)|_{r=r_0+\sqrt{\kappa}(\eta-\eta_0)} = 0; \Leftrightarrow \Psi'(r, \eta)|_{r=r_0+\sqrt{\kappa}(\eta-\eta_0)} = 0. \quad (38)$$

In this case in consequence of the equations for perturbations out of the boundary of the sound horizon the perturbations of energy density, pressure and velocity must automatically turn into zero:¹

$$\delta\varepsilon(r, \eta)|_{r>r_0+\sqrt{\kappa}(\eta-\eta_0)} = 0; \quad v(r, \eta)|_{r>r_0+\sqrt{\kappa}(\eta-\eta_0)} = 0. \quad (39)$$

An important private case of the boundary conditions (37) and (38) are the conditions at “the zero sound horizon “

$$\Sigma_0 : \quad r = \sqrt{\kappa}\eta. \quad (40)$$

In this case instead of (37) and (38) we have

$$\Psi(r, \eta)|_{r=\sqrt{\kappa}\eta} = \mu(\eta); \quad \Psi'(r, \eta)|_{r=\sqrt{\kappa}\eta} = 0. \quad (41)$$

¹ At the very boundary of the sound front the energy density perturbations and the velocities may practically have final breaks.

2.2 Solutions with zero boundary conditions at zero sound

Let us study some private solutions in the form of power series by radial variable satisfying the boundary conditions (41). Such perturbations can be generated by the metrics fluctuations at zero moment of time and concentrated at this moment of time in zero volume. Coming to the most convenient radial variable ϱ

$$r = \sqrt{\kappa}\rho, \quad (= \frac{1}{\sqrt{3}}\rho), \quad (42)$$

in the case of $\kappa = 1/3$ let us write down the field equations (6) and (7). The dot denotes derivatives by anew temporal variable

$$\ddot{\mu} + \frac{2}{\eta}\dot{\mu} - \frac{2\mu}{\eta^2} = 0, \quad (43)$$

$$\ddot{\Psi} + \frac{2}{\eta}\dot{\Psi} - \frac{2\Psi}{\eta^2} - \Psi'' = 0. \quad (44)$$

then solving

$$\mu = \mu_+\eta + \mu_-\eta^{-2}, \quad (45)$$

where μ_- and μ_+ are arbitrary constants.

N=3

As far as the Taylor-series expansion of the function Ψ should not consist of the zero power r by definition and therefore all the even powers in this expansion automatically vanish, so setting further

$$\Psi(\rho, \eta) = \Psi_1(\eta)r + \Psi_3(\eta)r^3 \quad (46)$$

and dividing the variables in the equation (44) we get an equation for the functions $\Psi_i(\eta)$:

$$\ddot{\Psi}_1 + \frac{2}{\eta}\dot{\Psi}_1 - \frac{2\Psi_1}{\eta^2} = 6\Psi_3; \quad (47)$$

$$\ddot{\Psi}_3 + \frac{2}{\eta}\dot{\Psi}_3 - \frac{2\Psi_3}{\eta^2} = 0. \quad (48)$$

In compliance with (45) we find from (48)

$$\Psi_3 = C_3^+ \eta + C_3^- \eta^{-2}, \quad (49)$$

where C_{\pm}^3 are some constants? Substituting the solution (49) into the right part of the equation (47) and defining private solutions of the obtained equation we shall find its general solution

$$\Psi_1 = C_1^+ \eta + C_1^- \eta^{-2} + \frac{3}{5}C_3^+ \eta^3 - 3C_3^-. \quad (50)$$

Thus,

$$\Psi = (C_1^+ \eta + C_1^- \eta^{-2} + \frac{3}{5} C_3^+ \eta^3 - 3 C_3^-) r + (C_3^+ \eta + C_3^- \eta^{-2}) r^3. \quad (51)$$

Further calculating $\Psi(\rho, \eta)|_{r=\eta} \equiv \Psi(\eta, \eta)$, substituting the result into the boundary condition (41) and setting equal the coefficients in the obtained equation by simultaneous powers η we shall get a set of equations for the constants C_i^\pm, μ_\pm

$$\left. \begin{array}{l} \eta^{-2} \\ \eta^{-1} \\ \eta \\ \eta^2 \\ \eta^4 \end{array} \right| \begin{array}{l} 0 \\ C_1^- \\ -2C_3^- \\ C_1^+ \\ \frac{8}{5}C_3^+ \end{array} = \begin{array}{l} \mu_-; \\ 0; \\ \mu_+; \\ 0; \\ 0. \end{array} \quad (52)$$

Thus, there are only two nonzero constants included into the expression for ν : μ_+ and $C_3^- = -1/2\mu_+$. So, finally

$$\Psi(\rho, \eta) = \left\{ \frac{3}{2}\mu_+\rho - \frac{1}{2}\mu_+\frac{\rho^3}{\eta^2}, -\rho\eta; -\mu_+\eta, -\rho > \eta \Rightarrow \right. \quad (53)$$

$$\Phi(\rho, \eta) = \left(\frac{3}{2}\mu_+\rho - \frac{1}{2}\mu_+\frac{\rho^3}{\eta^2} \right) \chi(\eta - \rho), -$$

It is the obtained previously retarding solution with zero boundary conditions at zero sound horizon [2] and $\chi(z)$ is the Heavyside function. At that supposing the following for the ultrarelativistic equation of state

$$a(\eta) = \eta; \quad (\kappa = 1/3), \quad (54)$$

we obtain

$$\nu(\rho, \eta) = \left(3\frac{\mu_+}{\eta} - \mu_+\frac{\rho^2}{\eta^3} - 2\frac{\mu_+}{\rho} \right) \chi(\eta - \rho), \quad (55)$$

It is easy to see that in this case at the surface of zero sound front $\rho = \eta$ the boundary conditions (41) are fulfilled identically.

N=5

Now let us study the fifth degree multinomial as a solution. In this case instead of the relations (44) and (45) we have

$$\Psi(\rho, \eta) = \Psi_1(\eta)\rho + \Psi_3(\eta)\rho^3 + \Psi_5(\eta)\rho^5; \quad (56)$$

$$\ddot{\Psi}_1 + \frac{2}{\eta} \dot{\Psi}_1 - \frac{2\Psi_1}{\eta^2} = 6\Psi_3; \quad (57)$$

$$\ddot{\Psi}_3 + \frac{2}{\eta} \dot{\Psi}_3 - \frac{2\Psi_3}{\eta^2} = 20\Psi_5; \quad (58)$$

$$\ddot{\Psi}_5 + \frac{2}{\eta} \dot{\Psi}_5 - \frac{2\Psi_5}{\eta^2} = 0. \quad (59)$$

Similarly to the previous case we have

$$\Psi_5 = C_5^+ \eta + C_5^- \eta^{-2}. \quad (60)$$

Substituting (60) into the right part of the equation (58), solving it similarly to the previous case and substituting the obtained solution into the equation (57) we get finally

$$\Psi_3 = C_3^+ \eta + C_3^- \eta^{-2} + 2C_5^+ \eta^3 - 10C_5^-; \quad (61)$$

$$\Psi_1 = C_1^+ \eta + C_1^- \eta^{-2} + \frac{3}{5}C_3^+ \eta^3 - 3C_3^- + \frac{3}{7}C_5^+ \eta^5 - 15C_5^- \eta^2; \quad (62)$$

and thus

$$\begin{aligned} \Psi(\rho, \eta) = & (C_1^+ \eta + C_1^- \eta^{-2} + \frac{3}{5}C_3^+ \eta^3 - 3C_3^- + \frac{3}{7}C_5^+ \eta^5 - 15C_5^- \eta^2)r + \\ & + (C_3^+ \eta + C_3^- \eta^{-2} + 2C_5^+ \eta^3 - 10C_5^-)r^3 + (C_5^+ \eta + C_5^- \eta^{-2})r^5. \end{aligned} \quad (63)$$

Substituting the equation (63) into the boundary conditions (41) and equating the coefficients under equal η degrees we get equations for the constants C_i^\pm, μ_\pm

$$\begin{array}{l|l} \eta^{-2} & 0 \\ \eta^{-1} & C_1^- \\ \eta & -2C_3^- \\ \eta^2 & C_1^+ \\ \eta^3 & -24C_5^- \\ \eta^4 & \frac{8}{5}C_3^+ \\ \eta^6 & \frac{24}{7}C_5^+ \end{array} \quad \begin{array}{l} = \mu_-; \\ = 0; \\ = \mu_+; \\ = 0; \\ = 0; \\ = 0; \\ = 0. \end{array} \quad (64)$$

Thus,

$$C_5^\pm = 0, \quad (65)$$

and the rest constants values coincide with the obtained ones above. It means that the solution coincides with the obtained one above. It is easy to show, adding of any new members of the series does not change the situation.

Thus, we have proved the theorem:

Theorem 1. *The only spherically-symmetric solution of the C^1 class of linearized around Friedmann space-plane solution to the Einstein equations for an ideal ultrarelativistic fluid, corresponding to the zero boundary conditions at the zero sound horizon (41), is the solution (53) (it is equivalent to (55)).*

2.3 Investigation of the retarding solution

From the formula (55) one can immediately see continuity of not only the first radial derivatives but also the first temporal ones of the $\delta\nu$ metrics perturbation and the potential functions

$$\left. \frac{\partial \delta\nu(\rho, \eta)}{\partial \rho} \right|_{\rho=\eta} = 0; \quad (66)$$

$$\left. \frac{\partial \delta\nu(\rho, \eta)}{\partial \eta} \right|_{\rho=\eta} = 0; \quad (67)$$

In Fig.1 and 2 graphs of temporal evolution of the potential functions $\Phi(\rho, \eta)$ and $-\delta\nu(\rho, \eta)$ are shown

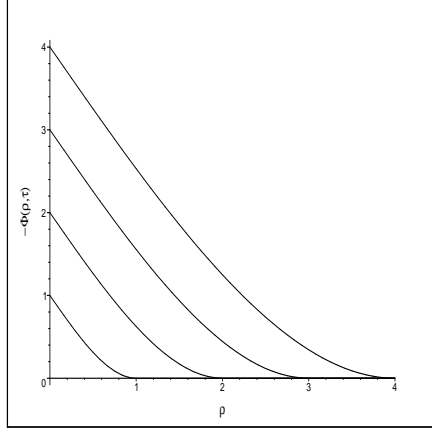


Fig.1. Evolution of the potential function $-\Phi(\rho, \eta)$. Bottom-up $\eta = 1; 2; 3; 4$. Along the abscissa axis the radial variable values $\rho = r\sqrt{3}$ are laid off.

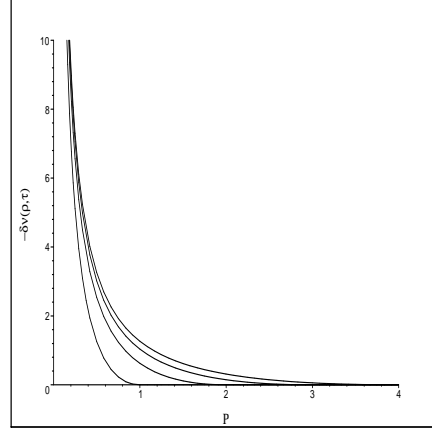


Fig 2. Evolution of the metrics perturbation $-\delta\nu(\rho, \eta)$. Bottom-up $\eta = 1; 2; 3; 4$. Along the abscissa axis the radial variable values $\rho = r\sqrt{3}$ are laid off.

The second radial derivatives of the potential functions and metrics have a final break at the sound horizon. In consequence of this the energy density perturbation has a final break at the sound horizon also. Calculating the relative energy density of the spherical perturbations according to the formula (11) we find

$$\frac{\delta\varepsilon}{\varepsilon_0} = -\frac{3\sqrt{3}\mu_+}{4\pi\rho} \left(\frac{\rho^3}{\eta^3} + 3\frac{\rho}{\eta} - 1 \right). \quad (68)$$

The jump of the relative energy density at the sound horizon is

$$\Delta = \left. \frac{\delta\varepsilon}{\varepsilon_0} \right|_{\rho=\eta} = -\frac{9\sqrt{3}\mu_+}{4\pi\eta} \quad (69)$$

and it decreases by the time (see Fig. 3). Let us turn to the formula for the radial velocity (12). Substituting the equation for Φ into this formula and coming to the radial variable ρ we obtain

$$v = -\frac{9\sqrt{3}\mu_+\eta}{16\pi\rho^3} \left(1 + 2\frac{\rho^3}{\eta^3}\right). \quad (70)$$

Evolution of the perturbation radial velocity is shown in Fig.4.

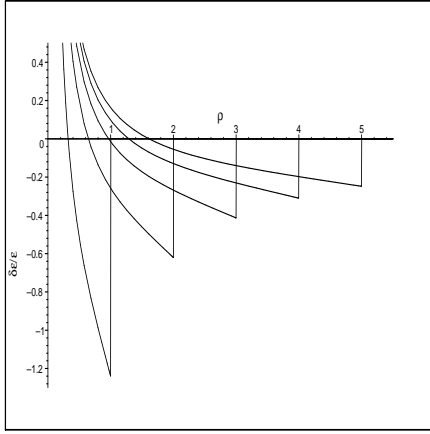


Fig.3. Evolution of the relative perturbation of the energy density $\delta\varepsilon/\varepsilon_0$. From left to right $\eta = 1; 2; 3; 4; 5$. Along the abscissa axis the radial variable values $\rho = r\sqrt{3}$ are laid off.

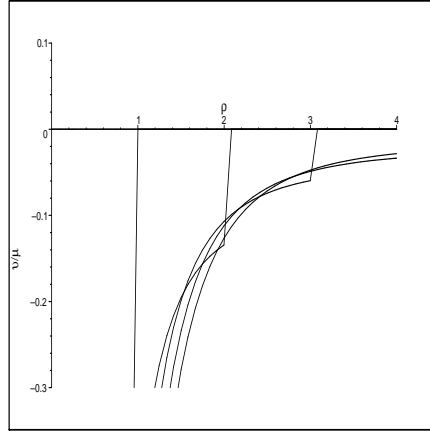


Fig 4. Evolution of the given radial velocity $v(\rho, \eta)/\mu_+$. From left to right $\eta = 1; 2; 3; 4; 5$. Along the abscissa axis the radial variable values $\rho = r\sqrt{3}$ are laid off.

3 Conclusion

Summarizing let us point out the following. It is easy to see that the obtained general retarding solution with a central singular source coincides with the earlier obtained ones [2],[3],[4] and [5]. However, note that in the mentioned works the solutions (53)-(55) were pointed out as private retarding solutions. The obtained retarding solutions belong to the class C^1 , however the second derivatives by radial variable have a break of the first genus at the sound horizon. As a result the energy density break at the sound horizon corresponds to the obtained retarding solution. Because of the pointed out conditions to find out the nature of the density break it is necessary to solve two tasks: to find the retarding solution for the barotrope arbitrary coefficient and solve the Cauchy task for the initially localized perturbation. We shall solve these tasks in our next papers.

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